

# On vanishing of the factorizable $c\bar{c}$ contribution in the radiative $B \rightarrow K^*\gamma$ decay

Dmitri Melikhov\*

*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120, Heidelberg, Germany*

We argue that the *factorizable*  $c\bar{c}$  long- and short-distance contributions to the  $B \rightarrow K^*\gamma$  amplitude vanish, separately, if defined in a gauge-invariant way. Therefore, the  $c\bar{c}$  states contribute to the radiative decay only through the *non-factorizable* soft-gluon exchanges.

The understanding of the long-distance effects in rare  $B$ -decays is an important theoretical problem. In the exclusive radiative decay  $B \rightarrow K^*\gamma$  there are two types of the long-distance contributions: (i) contributions due to the electromagnetic penguin bilinear quark operator, and (ii) contributions induced by the four-quark operators in the effective hamiltonian.

The contribution (i) is relatively simple and is described by the  $B \rightarrow K^*$  form factor. The four-quark operators (ii) lead to several contributions of the various types [1]: contributions of the intermediate  $c\bar{c}$  continuum and bound ( $\psi, \psi', \dots$ ) states; the weak annihilation; and other more complicated effects.

The contribution of the  $c\bar{c}$  states is of great importance for the semileptonic  $B \rightarrow (K, K^*)l^+l^-$  decay: the  $c\bar{c}$  vector resonances appear in the physical region at  $q^2 = M_{\text{res}}^2$ ,  $q$  the momentum of the lepton pair. There are various models [2–4] describing the  $c\bar{c}$  contribution as function of  $q^2$  based on the factorization [5].

The  $c\bar{c}$  states also influence the amplitude of the radiative  $B \rightarrow K^*\gamma$  decay which occurs at  $q^2 = 0$ . The latter is the only exclusive rare decay observed at the experiment, and its reliable theoretical description is strongly needed. The aim of this letter is to point out the following:

The gauge invariance requires that the full *factorizable*  $c\bar{c}$  contribution (the sum of the long- and short-distance contributions) to the  $B \rightarrow K^*\gamma$  vanish at  $q^2 = 0$ . If defined in a gauge-invariant way, the *factorizable* long- and short-distance contributions vanish, separately.

The  $c\bar{c}$  states in the  $q^2$  channel therefore contribute to the amplitude of the *real* photon emission only through the *non-factorizable* soft-gluon exchanges. The *factorizable*  $c\bar{c}$  contributions are essential for the emission of the *virtual* photon.

The amplitude describing the rare radiative decay reads

$$A = \langle K^*\gamma | H_{eff}(b \rightarrow s) | B \rangle, \quad (1)$$

where [6]

$$H_{eff}(b \rightarrow s) = \frac{G_F}{\sqrt{2}} \xi_t C_{7\gamma} \mathcal{O}_{7\gamma} - \frac{G_F}{\sqrt{2}} \xi_c (C_1(\mu) \mathcal{O}_1 + C_2(\mu) \mathcal{O}_2) \quad (2)$$

with  $G_F$  the Fermi constant,  $\xi_q = V_{sq}^* V_{bq}$ ,  $C_i$  the short-distance Wilson coefficients and  $\mathcal{O}_i$  the basis operators

$$\mathcal{O}_{7\gamma} = \frac{em_b}{8\pi^2} \bar{d}_\alpha \sigma_{\mu\nu} (1 + \gamma_5) b_\alpha F_{\mu\nu}^{(\gamma)}, \quad (3)$$

$$\begin{aligned} \mathcal{O}_1 &= \bar{s}_\alpha \gamma_\nu (1 - \gamma_5) c_\alpha \bar{c}_\beta \gamma_\nu (1 - \gamma_5) b_\beta, \\ \mathcal{O}_2 &= \bar{s}_\alpha \gamma_\nu (1 - \gamma_5) c_\beta \bar{c}_\beta \gamma_\nu (1 - \gamma_5) b_\alpha. \end{aligned} \quad (4)$$

The operator  $\mathcal{O}_7$  leads to the penguin form factor  $T_1^{B \rightarrow K^*}(0)$  known quite well [7]. We shall discuss the contribution of the 4-fermion operators  $\mathcal{O}_{1,2}$ , i.e. the amplitude

$$A \sim \langle K^*\gamma | \bar{s}_\alpha \gamma_\nu (1 - \gamma_5) b_\alpha \bar{c}_\beta \gamma_\nu (1 - \gamma_5) c_\beta | B \rangle. \quad (5)$$

Assuming factorization [5] this amplitude takes the form

$$A^{fact} \sim a_2 \langle K^* | \bar{s}_\alpha \gamma_\nu (1 - \gamma_5) b_\alpha | B \rangle \langle \gamma | \bar{c}_\beta \gamma_\nu (1 - \gamma_5) c_\beta | 0 \rangle, \quad (6)$$

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\*Alexander-von-Humboldt fellow. On leave from *Nuclear Physics Institute, Moscow State University, 119899, Moscow, Russia*

where  $a_2 = C_1 + C_2/N_c$ . The  $B \rightarrow K^*$  amplitude in this expression is given in terms of the known  $B \rightarrow K^*$  weak form factors. The photon amplitude  $\langle \gamma | \bar{c} \gamma_\nu (1 - \gamma_5) c | 0 \rangle$  contains contributions of the  $c\bar{c}$  states, both resonances and continuum. There were attempts to model the contribution of the  $c\bar{c}$  resonances to the photon amplitude and to estimate in this way the long-distance effects in the  $B \rightarrow K^* \gamma$  decay [1]. We are going to show however that the long-distance contribution to this amplitude vanish (as well as the short-distance one) if defined in a gauge-invariant way.

We start our discussion with the photon amplitude from Eq. (6). One finds

$$\langle \gamma | \bar{c} \gamma_\nu (1 - \gamma_5) c | 0 \rangle = e \epsilon_\mu(q) \Pi_{\mu\nu}^{c\bar{c}}(q), \quad (7)$$

where

$$\Pi_{\mu\nu}^{c\bar{c}}(q) = i \int dx e^{iqx} \langle 0 | T(\bar{c} \gamma_\mu c(x), \bar{c} \gamma_\nu c(0)) | 0 \rangle \quad (8)$$

is the charm contribution to the polarization of the vacuum. The conservation of the charm vector current  $\partial_\mu(\bar{c} \gamma^\mu c) = 0$  leads to the transversity of  $\Pi_{\mu\nu}^{c\bar{c}}$  such that it takes the form

$$\Pi_{\mu\nu}^{c\bar{c}}(q) = i (g_{\mu\nu} - q_\mu q_\nu / q^2) \Pi^{c\bar{c}}(q^2). \quad (9)$$

Let us follow the analysis of the long-distance  $c\bar{c}$  contributions of Refs [1,2]: The function  $\Pi^{c\bar{c}}(q^2)$  contains poles at  $q^2 = M_n^2$ , where  $M_n$  is the mass of the  $c\bar{c}$  vector resonance ( $\psi_n = \psi, \psi', \dots$ ), and the contribution of the  $c\bar{c}$  continuum. Neglecting the resonance widths we obtain in the region  $q^2 \simeq M_n^2$

$$\Pi_{\mu\nu}^{c\bar{c}}(q) = -i M_n^2 f_n^2 \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{M_n^2} \right) \frac{1}{M_n^2 - q^2} + \text{regular terms}, \quad (10)$$

where  $f_n$  is the leptonic decay constant of the vector resonance defined as follows

$$\langle 0 | \bar{c} \gamma_\mu c | \psi_n \rangle = \epsilon_\mu^{(n)} f_n M_n. \quad (11)$$

Then one calculates the contribution of the individual resonance to the factorized  $B \rightarrow K^* \gamma$  amplitude of Eq (6), takes a sum over all  $c\bar{c}$  resonances and obtains in this way  $A_{LD}^{fact}$ . The latter is given in terms of the  $B \rightarrow K^*$  weak transition form factors at  $q^2 = 0$  (see definitions in [7]):

$$\begin{aligned} A_{LD}^{fact} &= \sum_n A_n^{fact}(B \rightarrow K^* \psi_n \rightarrow K^* \gamma) \\ &= \frac{2}{3} e \frac{G_F}{\sqrt{2}} a_2 \frac{\sum_n f_n^2}{M_B + M_{K^*}} \epsilon_{(\gamma)}^{*\mu} \epsilon_{(K^*)}^{*\nu} \{ i \epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta + g_{\mu\nu} A_1(0) (M_B + M_{K^*})^2 + P_\mu q_\nu A_2(0) \}, \end{aligned} \quad (12)$$

where  $P = p_B + p_{K^*}$ ,  $q = p_B - p_{K^*}$ ,  $\epsilon$  the polarization vectors. Following [1,2]  $A_{LD}^{fact}$  is expected to describe the long-distance contribution to the amplitude of the radiative decay.

Clearly, this amplitude (as well as the contribution of the individual resonance) is not gauge-invariant if the form factors do not satisfy the relation

$$A_1(0) = -\frac{M_1 - M_2}{M_1 + M_2} A_2(0). \quad (13)$$

There were arguments that this relation is approximately satisfied in the leading order of the large-energy limit [1]. This does not help much, since one needs the relation (13) to be exact. But the  $B \rightarrow K^*$  form factors have no reason to satisfy this relation precisely, and therefore both  $A_n^{fact}$  and  $A_{LD}^{fact}$  are not gauge-invariant.<sup>1</sup> This means that the amplitude  $A_{LD}^{fact}$  of Eq. (12) is a gauge-dependent quantity and has no clear physical interpretation. In particular, it cannot be used as an estimate of the long-distance effects in the radiative decay (cf. [1]).

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<sup>1</sup> This becomes even more obvious when nonzero  $q^2$  are considered: in this case transversity of the resonance amplitude requires exact relations between the form factors valid for all  $q^2$ . Clearly, the form factors do not satisfy such relations.

Let us understand better the origin of the difficulty:

We consider the contribution of the resonance to the gauge-invariant amplitude  $\Pi_{\mu\nu}^{c\bar{c}}$ . Clearly, this contribution is gauge-invariant near the pole where the resonance dominates the amplitude.

Far from the pole, however, one can describe the individual resonance contribution in different ways: it is one of the many regular contributions to the amplitude. As one of the possibilities, the contribution of the resonance *far* from the pole can be defined in a gauge-dependent way - and that is what happened in the example above.

Obviously, this is allowed: nothing prevents us from splitting the gauge-invariant amplitude into many gauge-dependent parts. The only requirement is that the full amplitude - in our example the sum of the resonance and continuum  $c\bar{c}$  states - is gauge invariant. Working with such gauge-dependent parts is however inconvenient and can lead to a confusion in the interpretation of the results. Much better way is to define the contribution of the individual resonance in an explicitly gauge-invariant way.

The direct consequence of the gauge invariance is the relation

$$\Pi^{c\bar{c}}(q^2 = 0) = 0. \quad (14)$$

This is a well-known property which corresponds to the non-renormalizability of the photon mass, and this is an exact relation. Therefore it is convenient to work with the spectral representation for the polarization operator which requires a subtraction according to (14)

$$\Pi^{c\bar{c}}(q^2) = \frac{q^2}{\pi} \int \frac{ds}{(s - p^2)s} \text{Im } \Pi^{c\bar{c}}(s). \quad (15)$$

The imaginary part contains contributions of the resonances and the continuum states

$$\text{Im } \Pi^{c\bar{c}}(s) = \sum_n f_n^2 \delta(s - M_n^2) + \text{Im } \Pi_{cont}^{c\bar{c}}(s). \quad (16)$$

The expressions (15) and (16) for  $\Pi^{c\bar{c}}(q^2)$  lead to an explicitly gauge-invariant contribution of the individual resonance to the  $B \rightarrow K^* \gamma^*(q^2)$  amplitude

$$\begin{aligned} \bar{A}_n^{fact}(B \rightarrow K^* \psi_n \rightarrow K^* \gamma^*) &\sim a_2 \left( \frac{f_n}{M_n} \right)^2 \frac{1}{M_B + M_{K^*}} \epsilon_{(\gamma)}^{*\mu} \epsilon_{(K^*)}^{*\nu} \\ &\times \{ i \epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta V(q^2) q^2 + (g_{\mu\nu} q^2 - q_\mu q_\nu) A_1(q^2) (M_B + M_{K^*})^2 + (P_\mu q^2 - q_\mu P q) q_\nu A_2(q^2) \}. \end{aligned} \quad (17)$$

This expression can be used to describe the resonance contribution for any  $q^2$ .<sup>2</sup> Most interesting for us is that  $\bar{A}_n^{fact} = 0$  for  $q^2 = 0$ .

So, the explanation looks as follows:

The gauge invariant requires  $\Pi^{c\bar{c}}(0) = 0$ , and as a result of this relation we find for  $q^2 = 0$

$$\sum_n A_n^{fact} + A_{continuum}^{fact} = 0. \quad (18)$$

If we do not take care about the gauge invariance, then each of these contributions, separately, is ambiguous and only their sum has the physical interpretation. If we define both contributions in a gauge-invariant way as given by (17) then each of them vanishes for the real photon emission.

Summarizing, we come to the following conclusion:

- The full *factorizable* contribution of the  $c\bar{c}$  states to the  $B \rightarrow K^* \gamma$  amplitude vanishes at  $q^2 = 0$  as the direct consequence of the gauge invariance. If defined in a gauge-invariant way, the long-distance (resonance) and short-distance contributions to the amplitude of the radiative  $B \rightarrow K^* \gamma$  decay, separately, also vanish.
- Thus, the contribution of the  $c\bar{c}$  states to the amplitude of the *radiative* decay is completely non-factorizable, as has been already noticed in the literature [9,10]. As a consequence, the long-distance contribution cannot be expressed in terms of the  $B \rightarrow K^*$  form factors at  $q^2 = 0$ , but requires other relevant quantities for the description of the  $B \rightarrow K^*$  amplitude. For example, in [9] the non-factorizable  $c\bar{c}$  contribution at  $q^2 = 0$  was expressed in terms of the matrix element of the new - quark-gluon-photon operator.

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<sup>2</sup>Following [8], one can multiply the (gauge-invariant) resonance contribution in Eq. (17) by the phenomenological constant  $\kappa$  to describe correctly the branching ratio  $BR(B \rightarrow \psi_n X \rightarrow l^+ l^- X) = BR(B \rightarrow \psi_n X) BR(\psi_n \rightarrow l^+ l^-)$ .

At  $q^2 \neq 0$  relevant for the semileptonic  $B \rightarrow (K, K^*)l^+l^-$  decay, the factorizable contributions of the  $c\bar{c}$  states do not vanish. A gauge-invariant modelling of this factorizable contribution of the individual  $c\bar{c}$  resonance applicable at any  $q^2$  was discussed.

We discussed the  $B \rightarrow K^*\gamma$  decay, but the same arguments apply to other weak radiative  $B \rightarrow V\gamma$  decays,  $V$  the vector meson.

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- [1] B. Grinstein and D. Pirjol, Phys. Rev. D **62**, 093002 (2000) and refs therein.
  - [2] N. G. Deshpande, J. Trampetic, K. Panose, Phys. Lett. B **214**, 467 (1988).
  - [3] F. Krüger and L. M. Sehgal, Phys. Lett. B **380**, 199 (1996).
  - [4] M. Ahmady, Phys. Rev. D **53**, 2843 (1996).
  - [5] M. Neubert and B. Stech, in *Heavy flavours II*, A. Buras, M. Lindner (eds), World Scientific, Singapore [hep-ph/9705292].
  - [6] B. Grinstein, M. B. Wise and M. J. Savage, Nucl. Phys. **B319**, 271 (1989).
  - [7] D. Melikhov, N. Nikitin, S. Simula, Phys. Rev. D **57**, 6814 (1998);  
D. Melikhov and B. Stech, Phys. Rev. D **62**, 014006 (2000) and refs therein.
  - [8] A. Ali, T. mannel, T. Morozumi, Phys. Lett. B **273**, 505 (1991).
  - [9] A. Khodjamirian, R. Rückl, G. Stoll, D. Wyler, Phys. Lett. B **402**, 167 (1997).
  - [10] D. Melikhov, N. Nikitin, S. Simula, Phys. Lett. B **430**, 332 (1998).